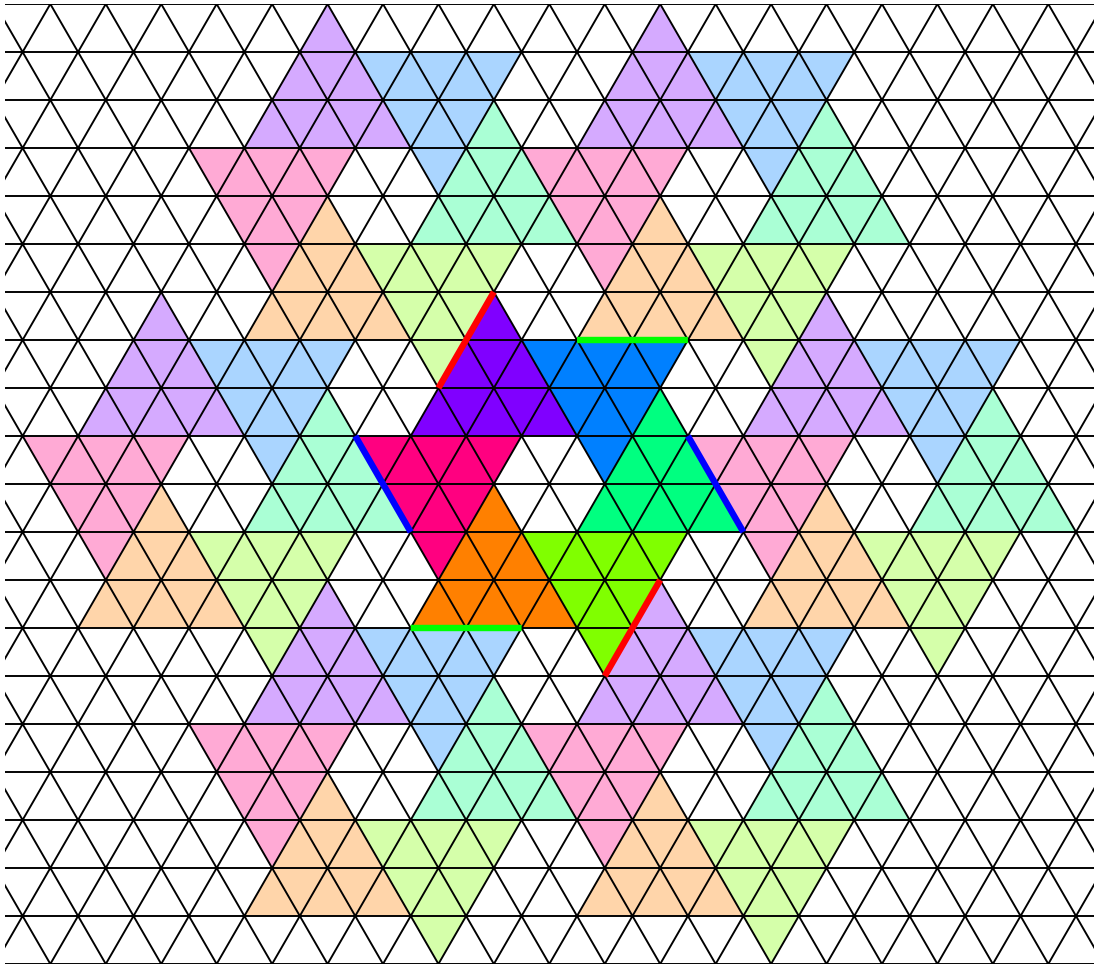
Flat Torus Star Sudoku

The central "star" in the diagram below (consisting of 6 different-colored triangles with 9 subtriangles each) is the diagram of a Star Sudoku puzzle. In a Star Sudoku puzzle, each triangular region, row, and slanted column contains the numbers 1 through 9. But wait, some of the rows and slanted columns don't have 9 adjacent cells. J.-P. Delahaye [1] addresses this by allowing rows and slanted columns, henceforth abbreviated RSC, to jump over the central hexagonal hole. This leaves 6 RSC with only 8 cells. Delahaye addresses this by making the nearby "point" of the star the ninth cell in each 8-cell RSC. I respectfully disagree. I believe that the Star Sudoku puzzle naturally lives on a flat torus, where the pairs of red, green, and blue edges in the diagram at right are identified, or glued together. The universal cover of this torus is a hexagonal tiling of the plane by copies of the central "star", extending the pattern shown here.



When we consider the Star Sudoku puzzle on the flat torus, we see that the RSC that appear to jump over the central hexagon actually extend around the outside through the colored edges, so that the 9 cells are in fact adjacent. Similarly, we see that the RSC with 8 cells extend through the colored edges to include a ninth cell. However, this is not the nearby point of the star but instead the diametrically opposite point of the star. The Star Sudoku puzzle in [2] satisfies the flat torus rules. The Star Sudoku puzzle in [1] satisfies Delahaye's rules instead, but can be made to comply with the flat torus rules by exchanging each of the three upper right points of the star with its immediate neighbor.

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References:

[1] "The Science Behind Sudoku", *Scientific American,* June 2006, pp. 80-87
[2] Sudoku Official Instructions Booklet, First World Sudoku Championship,
http://www.wsc2006.com/pdf/booklet_int.pdf